

# **VARIOUS EXPERIMENTAL APPLICATIONS OF DIGITAL IMAGE CORRELATION METHOD**

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## **Abstract**

Despite the powerful of current calculus codes by finite element or analytical, their reliability and validity must be justified by experimental tests. An original software for the displacement field measurement is presented in this paper. This software use the digital image correlation principle. Thanks to this technique, the precision of the measure could reach 1/60 th of a pixel and, it is possible to measure strains between  $5.10^{-5}$  and 0.8.

Initially, this software was developed for measuring strains on a sheet testing in metal forming. But, it is revealed that this application field is more important : biomechanical field, geotechnic field, metal characterization, control tests, ....

This software could be used, nowadays, for all application which need the knowledge of displacement and strain field for a plane surface. Of course, the experimental conditions must be in such a way that any deterioration of the random aspect, deposited on the piece, will occurs. Lost, alteration or little modification of the speckle aspect will not guarantee the success of the measure.

Finally, the purpose of this paper will be, first, to give clearly the mathematical principle of the displacement field determination by a correlation method. Then, various experimental tests using the correlation method, are presented. The powerful of this technique is show by their variety. This method of measure without contact can be an another tool for validating, in a short time, a numerical or analytical result.

## **1. Introduction**

The accurate knowledge, without contact, of displacement field and/or strain field for a plane body is more and more researched. The new technique introduced in this paper for the displacement and strain field measurement have in the same time a very large strain range measurement and also a good precision on the displacement measurement. Also, this technique reduces the computation time thanks to a displacement field determination entirely computerized. An continuous information field could be obtained with an important density of the measured point : more than 15000 points for a field of 1024 by 1024 pixels. This new technique requires only a speckle aspect which could be put on the specimen in few seconds with a paint spray.

## 2. Method

### 2.1. Bibliography

This technique was principally developed by M. A. Sutton et al [sut 83]. The following principles are used for a technique by digital image correlation. The image of the body could be represented by a discrete function : value between 0 and 255 of its grey levels. The correlation calculation are done for a group of pixels called pattern. The displacement field is supposed homogeneous inside the pattern. The initial image representing the body before distortion is a discrete function noted  $f(x,y)$  will be transformed in an other discrete function noted  $f^*(x^*,y^*)$  after distortion or displacement. The theoretical relation between the two discrete functions could be written as :

$$f^*(x^*,y^*) - f(x + u(x,y), y + v(x,y)) = 0 \quad (1)$$

$u(x,y)$  and  $v(x,y)$  represent the displacement field for a pattern. The figure 1 presents different used definitions :

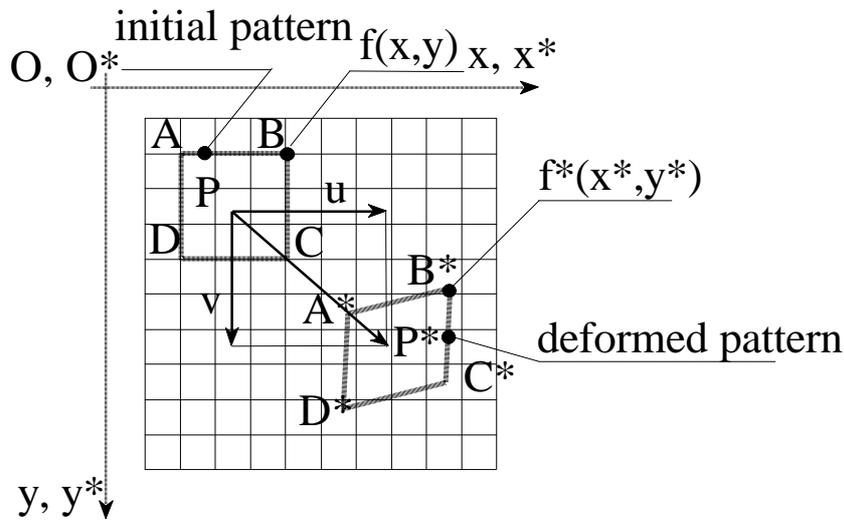


Figure n° 1 : Notations

For a best understanding, the initial and the deformed image are represented in the same axis.

The four essential element for a correlation technique are :

- the mathematical definition for the displacement field on a pattern which has to include, at the same time, the strain terms and the term of rigid body displacement,

- a mathematical correlation criterion between the two discrete functions  $f(x,y)$  and  $f^*(x^*,y^*)$ ,
- an interpolation method for the grey level of the images is needed in order to reach a sub pixel precision [pra 91], and,
- the mathematical solution for the determination of the elongation and the strain term for a pattern, [sut 86], [sut 83], [bru 89].

## 2.2. Mathematical principle of the method

### 2.2.1. Choice for the displacement field [mgu 96]

The presented method uses the following displacement field because the research of all its terms (rigid body, strains) will be done by a bilinear interpolation. So, the displacement field for a pattern is taken homogenous and bilinear in  $x$  et  $y$  as :

$$\begin{cases} u(x, y) = au \cdot x + bu \cdot y + cu \cdot x \cdot y + du \\ v(x, y) = av \cdot x + bv \cdot y + cv \cdot x \cdot y + dv \end{cases} \quad (2)$$

It contains the rigid body translation terms ( $du$  and  $dv$ ), the elongation terms ( $au$ ,  $av$  and  $bu$ ,  $bv$ ) and the shearing terms ( $cu$  and  $cv$ ).

### 2.2.2. Choice for the correlation coefficient

The most used correlation coefficient for a correlation technique are the least square and the crossed coefficients [chu 85] :

$$C_1 = \int_{\Delta M} (f(x, y) - f^*(x^*, y^*))^2 \cdot dx \cdot dy \quad (3)$$

and

$$C_2 = 1 - \frac{\int_{\Delta M} f(x, y) \cdot f^*(x^*, y^*) \cdot dx \cdot dy}{\sqrt{\int_{\Delta M} f(x, y)^2 \cdot dx \cdot dy \cdot \int_{\Delta M} f^*(x^*, y^*)^2 \cdot dx \cdot dy}} \quad (4)$$

Where,  $\Delta M$  represent the surface of the pattern in the initial image.

### 2.2.3. Research of the displacement field

The originality of this method is the following way of researching the displacement field. Consider a pattern centred in  $P$  and tops  $A$ ,  $B$ ,  $C$ ,  $D$  (figure n°1).

The correlation calculations will be done on the fours patterns centred in  $A$ ,  $B$ ,  $C$ , and  $D$  with the following supposition : their strain field are identical to the one of the pattern centred in  $P$ . These calculations are going to determine the rigid body displacement of the point  $A$ ,  $B$ ,  $C$ , and  $D$  which will be used to

obtain all the terms of the strain field for the pattern centred in P. The exact solution from a numeric view will be determined with the help of an iterative process : the rigid body displacement of the points A, B, C, and D will be searched during the iteration 'i' with the help of the component of the strain field obtained at the iteration '(i-1)'.

At the beginning of the iterative process, a rough initial solution for the two components of the displacement field is given for a pattern. The strain field inside this pattern centred in P is therefore reduced, at the first iteration, to the rigid body displacement terms only, because the strains terms are considered initially void. The newest position for the points A\*, B\*, C\*, and D\* in the final image are searched successively for this first iteration with the help of this strain field. This research is done by scanning following x and y. The accurate position of the new position of the points is obtained by the decreasing evolution simultaneously of the scan size zone and the scan steps. These displacements which allow to obtain a minimal correlation coefficient are noted  $du_{A1}$  and  $dv_{A1}$  for the point A and respectively,  $du_{B1}$ ,  $dv_{B1}$ ,  $du_{C1}$ ,  $dv_{C1}$ ,  $du_{D1}$ ,  $dv_{D1}$ , for the points B, C, D. With this eight values, the displacement field in P is obtained by a bilinear interpolation. The rigid body displacement calculation is continued until the following stability :

$$|du_{Pn} - du_{P(n-1)}| \leq \varepsilon \quad \text{and} \quad |dv_{Pn} - dv_{P(n-1)}| \leq \varepsilon \quad \text{with} \quad \varepsilon = 0.01 \text{ pixel} \quad (5)$$

In the case where the iterative process would not converge, it is stopped at the 10 th iteration and the pattern is considered aberrant and will be not be taken into account for the strain calculation. Then, an interpolation allows to find again the missing points after the displacement field calculations. The displacement field being obtained for this first pattern, the solution for the neighbours has to be searched in order to obtain the solution for the whole image. Consequently, all the patterns around this first pattern, will take as the initial solution, for the iterative process, his displacement field.

#### ***2.2.4. Research of the strain field***

The strains calculation could be done for one pattern or for a group of pattern. This calculation use the gradient of the transformation F and the Green-Lagrange tensor E [cam 94] :

$$[\mathbf{F}] = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \quad \text{and} \quad \mathbf{E} = \frac{1}{2} \cdot ({}^T\mathbf{F} \cdot \mathbf{F} - \mathbf{I}) \quad (6)$$

With this components, the principal strain are given by :

$$\begin{cases} \varepsilon_I = \ln(E_{11} + E_{22} + \sqrt{(E_{11} - E_{22})^2 + (2 \cdot E_{12})^2 + 1}) \\ \varepsilon_{II} = \ln(E_{11} + E_{22} - \sqrt{(E_{11} - E_{22})^2 + (2 \cdot E_{12})^2 + 1}) \end{cases} \quad (7)$$

The angle between the first strain main axis and the image axis x is given by :

$$\Phi = \text{Atn} \left[ \frac{2 \cdot E_{12} \cdot \sqrt{\frac{\ln(E_{11} + E_{22} + \sqrt{(E_{11} - E_{22})^2 + (2 \cdot E_{12})^2 + 1})}{\ln(E_{11} + E_{22} - \sqrt{(E_{11} - E_{22})^2 + (2 \cdot E_{12})^2 + 1})}}}{E_{11} - E_{22} + \sqrt{(E_{11} - E_{22})^2 + (2 \cdot E_{12})^2 + 1}} \right] \quad (8)$$

### 2.3. Computer implementation

From an initial and final image, the user could define one or several study zone. Only one operation is necessary to start the iterative process : give the first solution with the mouse. The algorithm is programmed in several languages : Fortran on a workstation, C and Fortran on a PC (Windows 3.11). Several result are available at the end of the calculations : the displacement field following x and y axis, the strain field following x and y axis, the deformed mesh and a vectorial representation of the displacement field.

## 3. Material

This correlation technique requires a special equipment and few precautions. The analyzed image must have the most random aspect as possible : each pattern must be different to another one. Usually, this speckle aspect is obtained by pulverization of white and black paint bomb. The images are catching with a numeric camera Kodak Mega Plus (1024\*1024 pixels). This camera high definition with joined pixels is linked to an acquisition card Matrox PIP1280 connected by a PC 486 DX50. Each image represent around 1 Mega Octet. The objective is a NIKON one (Micro-NIKKOR : 587910). A constant light during the taking of images has to be assured : cold light or ambient light. For information, the computation time on a Pentium 90 is 0.2 seconds for a pattern of 9 pixels by 9 pixels. This time reduces to 0.04 seconds for the same processing executed on a work station HP735.

## 4. Applications

The following applications are very various. They concern also the mechanic domain, the geotechnical domain, biomechanical.... The small and also the large strains are treated with the same software. The cover field vary from 7 x 7 mm<sup>2</sup> until several m<sup>2</sup>. The distance between the camera and the model (specimen, sheet, bone,....) could vary until several meters.

#### ***4.1. Strain field for a XES steel in his plastic domain under plane strain condition***

An application is done for the Renault Laboratory. The engineers drew a special specimen for a plane strain test. The specimen width is 15.3 mm, its length is 31 mm and its thickness is 1 mm. The figure \*\*\* shows the cover field by the CCD camera : 160\* 160 mm<sup>2</sup>. An initial image is held as reference and an another one for a certain load. After the treatment by correlation, the following results are obtained.

The deformed mesh is represented on the figure \*\*\*.

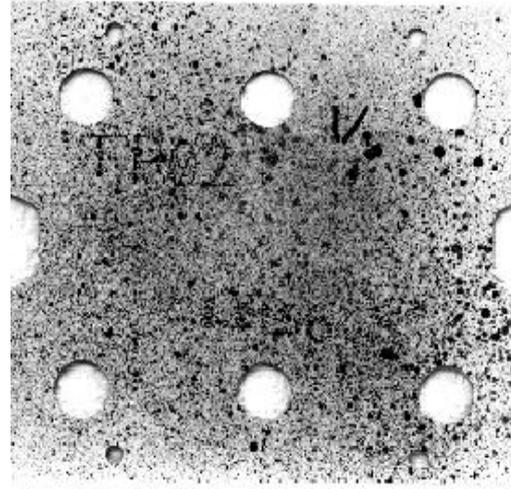


Figure \*\*\*: Specimen

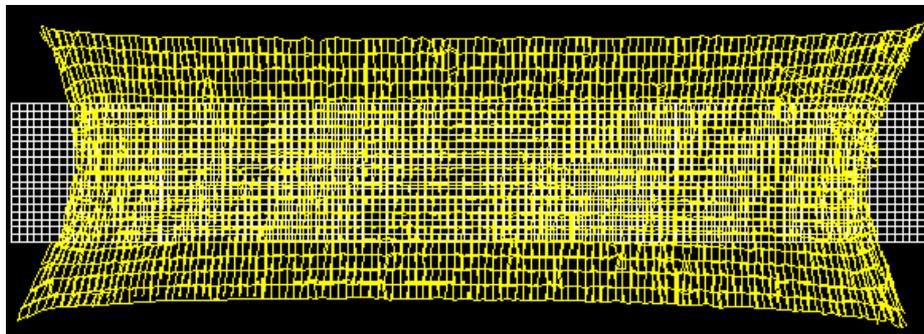


Figure \*\*\* : Deformed mesh x 20

A strain rate of 5% following the vertical axis is calculated in the specimen middle. The transverse strain distribution is reproduced on the figure \*\*\*.

This strain distribution underlines that the plane stress condition is achieved only on 30% of the total width. The advantages of this correlation technique for this special application are the following. First, an excellent precision on the result which reaches 6  $\mu\text{m}$ . Also, an important density of the measured point around 2124 give an continuous information.

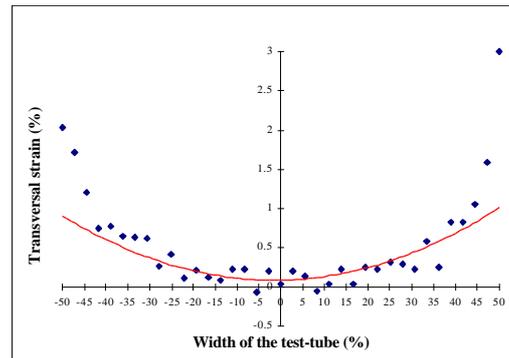


Figure \*\*\* : Transversal strain distribution

#### 4.2. Measure on a tunnel model

The Geotechnic Laboratory (Insa-Lyon) makes researches on the interaction ground-tunnel. So, the Laboratory have a simulation model for the ground behaviour around the tunnel.

The ground is replaced by "Schneebe" rollers : 150000 small cylinders of 4 mm diameter and \*\*\* mm height. They are placed "les uns sur les autres" on a width of 1500 mm and on a height of 1000 mm. The tunnel diameter could vary in order to simulate the compression and decompression phases. This model allows the calculation and the visualization of the analogical ground behaviour around the tunnel during the simulation of the digging phases. The figure \*\*\* show the experimental device.

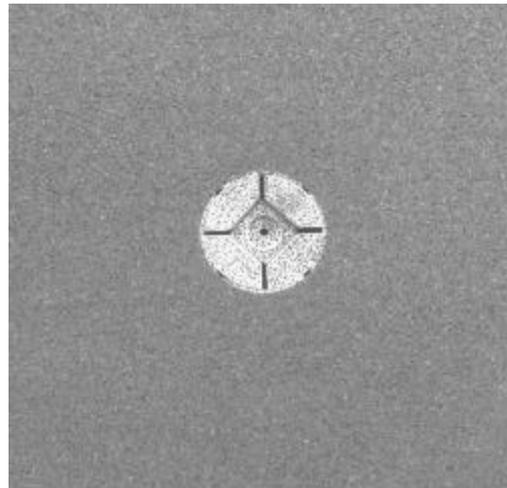


Figure \*\*\* : Experimental device

The CCD camera covers the totality field : 1000 mm by 1500 mm. The distance between the camera and the model is 15 m. The random aspect is obtained here in very simply manner : they are the "Schenbi" rollers which acts as speckle. Only, on the tunnel, initially uniform, some tasks were done by marker in order to assure the correlation on the whole image.

An initial image is captured as reference. Then, several images are considered for different levels of mandrel opening. After the correlation treatment between two images, various result are obtained. The deformed mesh informs immediately on the digging effect of a tunnel. The digging is shown by the figure \*\*\* which give the deformed mesh :

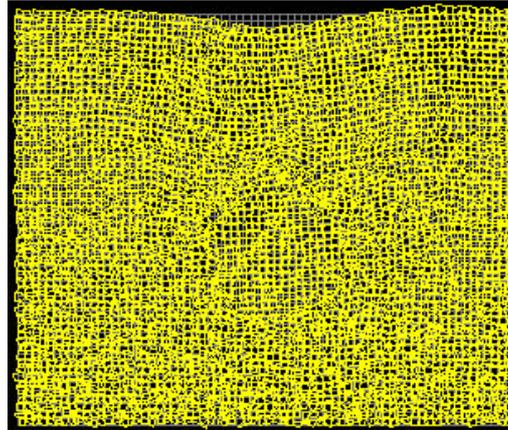


Figure \*\*\* : Deformed mesh

In this special application, the advantages of the correlation technique are various. The precision on the measure is very good : 50  $\mu\text{m}$  for a cover field of 1000 by 1500 mm. The number of measured point is very important : 11400 points.

#### ***4.3 Feasibility study on the strain field measurement for a specimen in bending after an impact***

A feasibility study was achieved for the CEA (Commissariat à l'Energie Atomique). Strains must be measured on a specimen tested in bending after an impact with the correlation method. Two photos are held : before and after the impact. These photos are next digitalized in order to be analyzed by correlation. The correlation calculations will be done only in the cranked section of the specimen. The following photo represents the specimen after the impact :

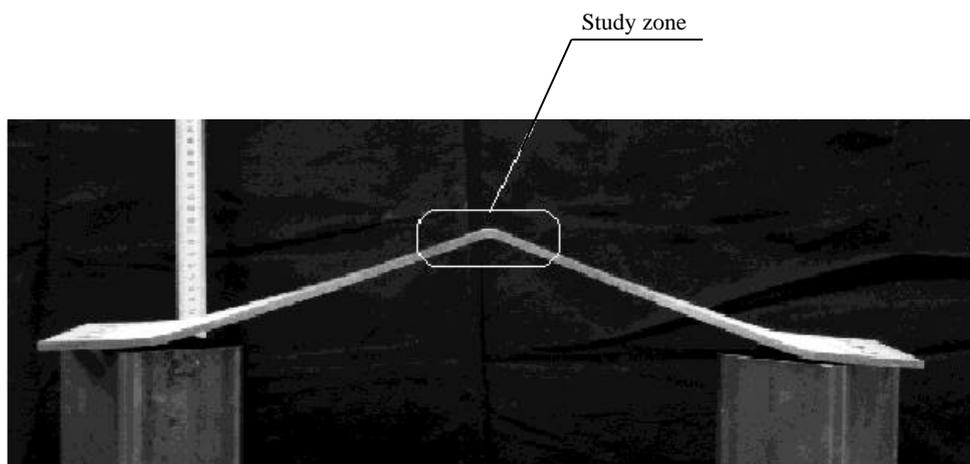


Photo : Specimen after the impact

On the cranked section of the specimen the out-of-plane motion is small so that the measured in plane deformation are accurate. The strain field following the V axis is reported on the figure \*\*\*. A strain rate around 48% is measured by this correlation method on the cranked section.

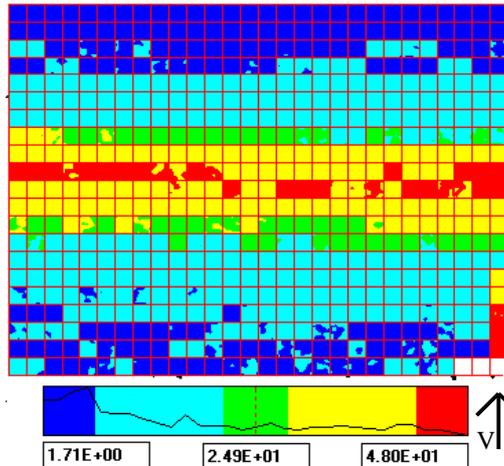


Figure : Strain field following the v axis obtained by correlation

#### 4.4. Determination of Forming Limit Curves by correlation

The Marciniak test [arr 90] is chosen for the experimental determination of Forming Limit Curves. Thanks to the hollow plane bottom of the punch, the necking will occurs on the plane surface of the sheet so a correlation technique will be easily used. In order to simplify the experimental device (servo control of the camera in order to maintain a stationary focal distance,...), a new module was developed by Schultze [sch 95]. This new module of metal forming was specially conceived for the correlation technique : a reverse module is achieved. A module with fixed punch and a mobile matrix is made.

The main purpose of this reverse is to have a camera and a stationary focal distance during the metal forming test. Also, the montage must also allow a sufficient lighting of the motif in order to obtain a good quality images, figure \*\*\*. During, the movement, the sheet was stamped on the fixed punch. The system originality is the use of an inclined mirror at 45° which restores the image on the camera placed front of the module. The new structure allows to stamped sheets until a maximal depth of 45 mm. Thanks to the correlation technique, the classical markings (grids,...) were not used.

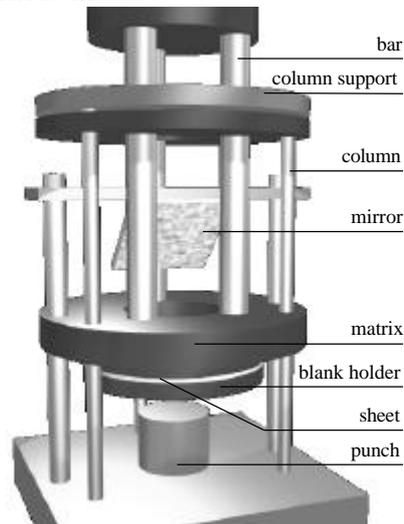


Figure \*\*\* : New metal forming module for the correlation technique

Now, a simple deposit of white paint then black by pulverisation in few second is sufficient. An other advantage of this technique correlation is that the step grid could be modified at the beginning of the calculations.

For the strain measurement on a sheet tested in metal forming, a pattern will be constituted by 8 to 12 pixels. In this configuration, a pixel is around 0.07 mm, so the grid size could vary from 0.59 mm to 0.88 mm. Sheets were cut following different width in order to cover all the strain path. Then, the Marciniak test is managed until the necking. The case of the uniaxiale traction is selected in order to illustrate the calculations. A study zone is held perpendicularly to the necking, figure\*\*\*. An initial image is captured as reference. Then, the Marciniak test is started. At the necking occurrence, an other image is held.

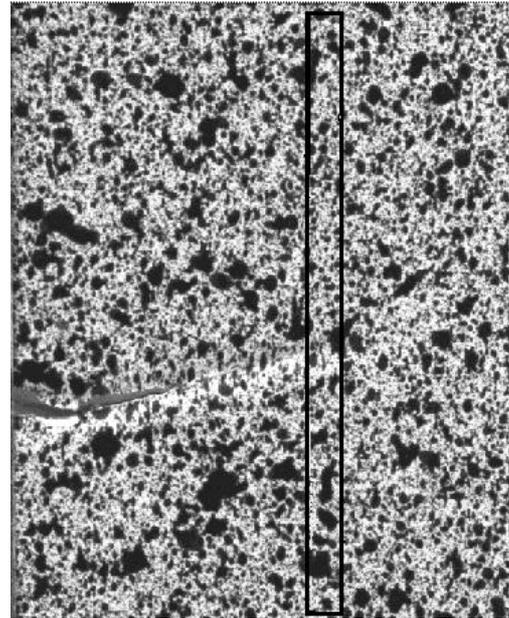


Figure \*\*\* : Study zone

Then, the Braggard interpolation [arr 90] is chosen for the measurement of the limit strain. The following Forming Limit Curve is then obtained by correlation for an aluminium (5754 mild finish) :

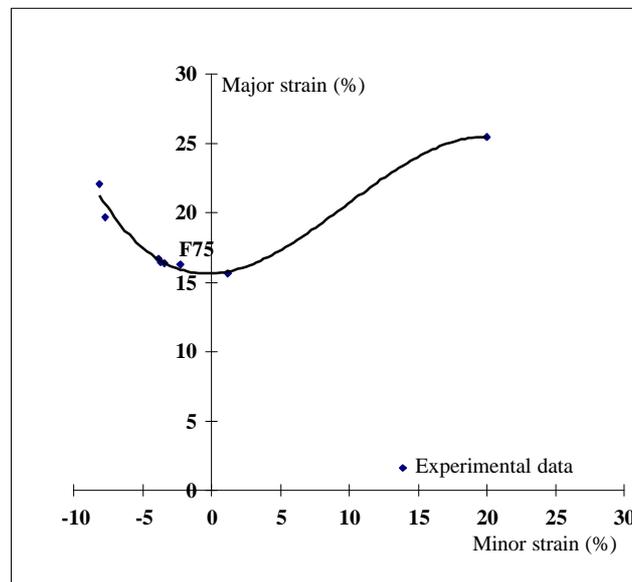


Figure \*\*\* : Forming Limit Curves for the aluminium 5754 mild finish obtained by correlation

Strain rate around 25% is measured by this correlation technique. Thereafter, the Forming Limit Curve for another materials were determined by correlation. A strain rate of 80% was appreciated for a XES steel between two images.

The advantages of this correlation technique for this application are various. First, there is the abandonment of classical marking, which represents a big saving of time. Then, the computation time are strongly reduced thanks to the correlation algorithm. Also, the precision on the limit strain is improved. The precision reaches  $0.5 \mu\text{m}$  on the displacements and \*\*\* on the strain.

#### ***4.5. Mechanic characterization of human spongy bone.***

The Biomechanical Laboratory (Insa-Lyon) manages a study touching the loss bone for aged humans. For this, it is necessary to measure mechanics characteristic. But, it is hard to stick a strain gauge on the bone.

So, a correlation method was tried. Here, the speckle aspect is obtained by a graphite pulverization on the bone. The sample size is a cube of 7 mm. A compression test is made. Then, several images are considered for different loads. After treatment, by the correlation software, the deformed mesh is obtained. The figure \*\*\* underlines the bone fracture according a privileged direction. The experiment confirmed this result. The precision on the result is about  $0.5 \mu\text{m}$ .



Figure \*\*\* : Deformed mesh

#### ***4.6. Traction-compression tests on "bielles"***

The Gec Alsthom company checks the unscrewing holding of "bielles" carbon/steel tested in alternating traction-compression cycles : the relative unscrewing has to be appreciated.

The tests are achieved on a traction-compression machine (Schenck). The relative unscrewing between the 'manchon' and the body of the "bielle" is measured, at the end of several cycles, with the correlation software. For each test, a reference image is held before the start of the cycle, then this image will be compared with a final image. For this configuration, a pixel represents 0.458 mm. The precision is around 1/100 millimetre. The steel 'manchons' and the body in carbon of the "bielle" was painted with black and white bomb paint : figure \*\*\*. The [bielle] is tested in traction-compression cycle and in traction. Each time, a relative unscrewing is found and measured. A unscrewing inferior to 1/10 millimetre is measured by this correlation technique.

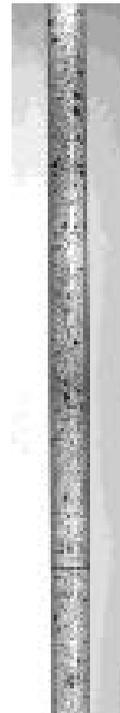


Figure \*\*\* : Bielle

#### ***4.7. Study of friction on a punch during a metal forming***

A thesis is proposed on the study of the friction on a punch during a metal forming [bur]. The Erichsen test is considered. The punch has a hemispherical form, the strain state is biaxial. The friction coefficient is directly bound to the strain distribution of the stamped sheet and to the strain on the pole of the stamped sheet. A pole is the first point in contact during the metal forming. An image is held as reference, then others during the metal forming. The figure \*\*\* shows a figure catches with a certain rate of strain : the pole is well show :

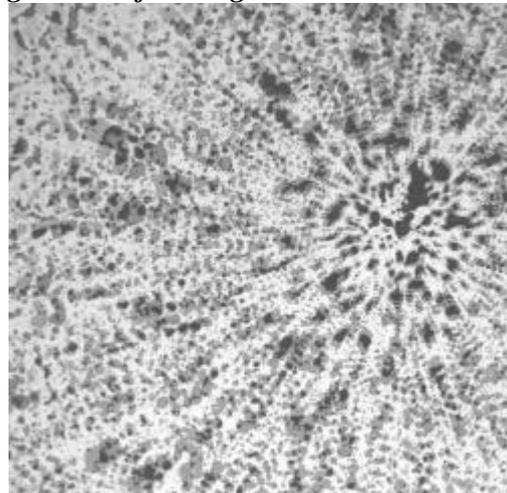


Figure \*\*\* : Figure catches with a certain rate of strain

The strain field gives a strain value of 5.23% following the vertical axis and 4.67% following the horizontal axis in the pole of the stamped sheet. The representation of the vector displacement, figure\*\*\*, show also the sheet pole. Because of the hemispherical form of the punch, the strain result, excepted for the pole, must be held with care (it was shown that a out of plane motion of 100 microns produces 232 micro strains).

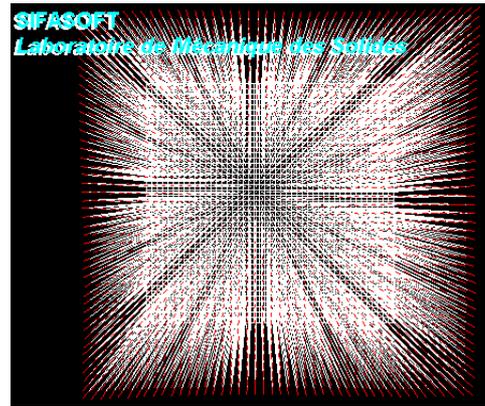


Figure \*\*\* : Displacement vectors

#### **4.8. Determination of the Lankford coefficient on a specimen in traction**

In order to take account the materials anisotropy, the Lankford coefficient are used. They are obtained by a traction test. If one notes  $x$  the transverse direction then the Lankford coefficient is written as :

$$r = \frac{d\varepsilon_x}{d\varepsilon_z} \quad (9)$$

So, with the incompressibility hypothesis :

$$r = -\frac{\varepsilon_x}{\varepsilon_x + \varepsilon_y} \quad (10)$$

The Lankford coefficient for a XES steel is found by the correlation technique. A standardized specimen with a 20 mm width is previously painted. Then, an other image after traction is held. Then, strains are calculated by correlation. For this special application of the determination of the Lankford coefficient, the computation time is very short because the correlation calculation will be done only on 4 small zones. The following result is found :

$$r = 2.54 \quad (11)$$

The most advantage for this application is : effect border parasite did not be taken in count for the measure which represents an improvement with regard to another manual method.

#### **4.9. Implantation study of prosthesis on calcaneum**

This study is achieved in collaboration with the "hôpital Sud de Grenoble". The purpose is to study the calcaneum, foot bone, and its fracture mechanisms. To cure different fractures of calcaneum, a "osteosynthèse" by "plaques vissée" of the calcaneum fractures is proposed. The camera is placed perpendicularly to the study plan, figure \*\*\*. Several images of the montage are considered for

different values of compression load, and a first image as reference. By correlation, the relative displacement of the fracture pieces is measured between different pieces.

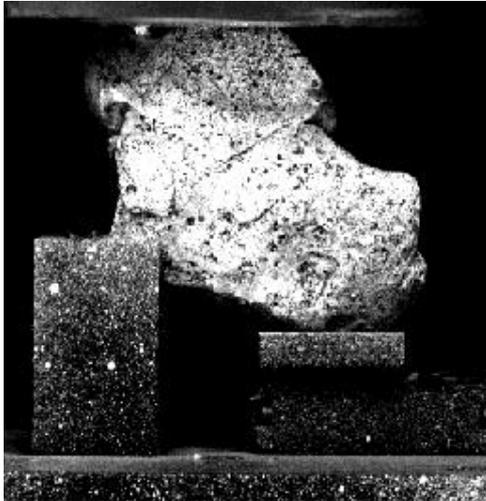


Figure \*\*\* : Astragale and calcaneum

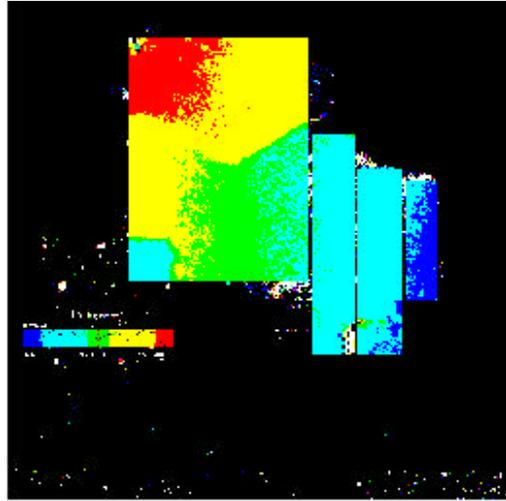


Figure \*\*\* : Deformed mesh

This technique allowed to know precious information on the fragments displacement of the fracture and consequently to make a choice between several "ostéosynthèses" or to bring modifications to them.

#### ***4.10. Study of an sensor for the biomechanical domain***

In the bioemechanical domain, some characteristic, as for example the young modulus must be determined for the human or animal bone. That is why, the following sensor was developed. Two gages sensor are put on the wings of the sensor in order to make the measure, figure \*\*\* :

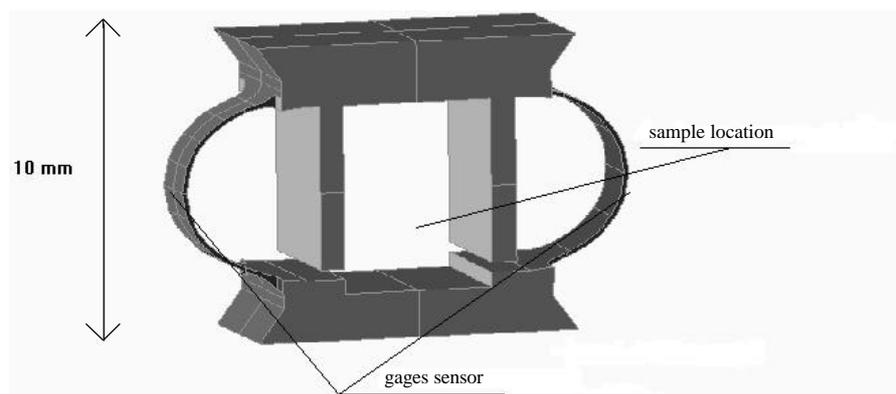


Figure \*\*\* : Sensor

In order to check this sensor, some tests are done, initially on aluminium, dural or plexiglas specimen. Their young modulus are known and the purpose is to recover these values by the sensor. But aberrant results are found. The

correlation technique is used here in order to see the sensor behaviour and also the specimen behaviour during the compression test. For this reason, the specimen and also the sensor were recovered by white and black paint. Then, a compression test under small load is realised. Two images are held and correlated.

The correlation calculations were done on the aluminium sample and on a band take on the sensor top. The deformed mesh is represented on the figure \*\*\*. One sees clearly that the sample and also the sensor distorts. The correction of the head side of the sensor are then achieved which improves the next results. Finally, it is concluded that this sensor could function only on materials which the young modulus is small like the bone one where the young modulus are ranges between 200 MPa (for the man) and 1500 MPa (for the sheep).

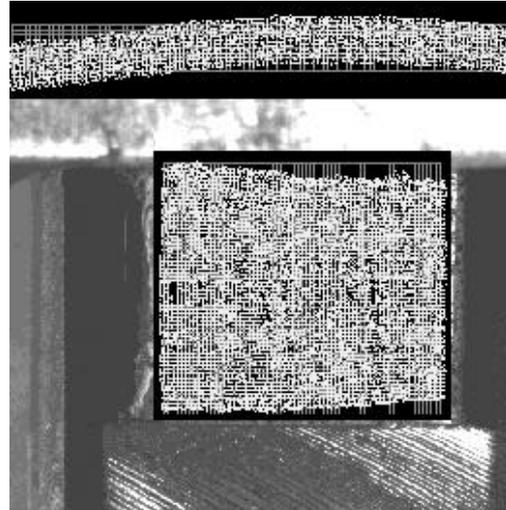


Figure \*\*\*: Deformed mesh X 100

#### ***4.14. Determination of the beds rivers displacements***

A thesis is suggested by the "region Rhône-Alpes" and the "Cemagref", and coordinated by this last one, entitled "Gestion ,écologique intégrée des Hydrosytèmes". The topic involves the "Laboratoire de Mécanique des Fluides de l'Insa de Lyon" and "le Laboratoire d'Ecologie des Eaux Douces et des Grands Fleuves et la Division Hydrologie-Hydraulique du Cemagref-Lyon". The different impact conditions of drainage and solid apports, on the structure and the dynamic of substrate has to be studied experimentally and also, the invertebrates actions on the beds dynamic, and in particular by a the sediment movement. A first study is achieved to decide the correlation technique efficiency for this special application. So, the following test is managed. Some fine sand and stones are arranged on a cloth in a random manner. An image is held as reference. Then, the cloth is infinitesimally displaced in order to simulate for example a sediment river motion. The second photo being captured, the correlation algorithm is applied. The following figure represents the displacement field following the \*\*\* axis obtained between the two states.

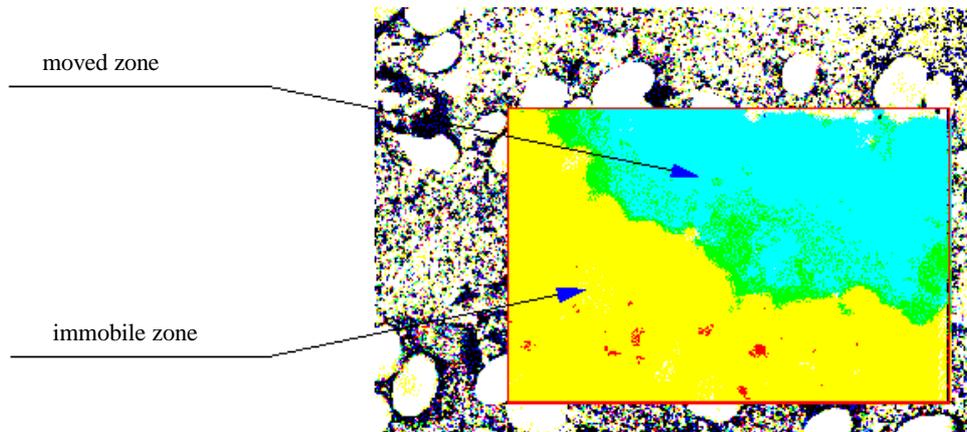


Figure \*\*\* : Displacement field following the "\*\*\*" axis

## 5. Discussion

Some points about the method utilization are underlined. First, we must ensure that the random aspect is not subjected to any modifications during the tests. A partial lost or a change of his aspect will carry a wrong correlation.

Afterwards, there is the lighting problem during the tests. A high contrast for the image is required. So, a suitable lighting of the study zone has to be set up. It has to assure, principally, an uniform lighting of the study zone.

A last point still to underline. This particular point touches directly the software utilisation and principally the different correlation parameters adjustment. Indeed, there are five main correlation parameters : the correlation coefficient ( $C_1$  or  $C_2$ ), the grey level interpolation (bilinear or bi cubic spleen), pattern size (from 6 x 6 pixels to 35 x 35 pixels), grid size (from 6 x 6 pixels to 35 x 35 pixels) and in last the speckle aspect (fine or coarse). The parameter adjustment must be done in function of different situations : small or large strains, required precision, computation time... The developed software allows to the user, during the put in data, to make the choice between this different parameters, the difficulty is to make the best choice. So it is necessary to know, by advance, the influence of each parameter on the precision, the computation time,...

## 6. Conclusion

The various tests described in this paper underlines all the interest that one has to carry on this correlation technique for the displacement field measurement. This is very powerful technique : strains until 80% were appreciated between two images.

Nevertheless, some researches still to be done. For example, the computation time must be reduced. More, it would be necessary to modify the correlation program in order to allow the choice of any study zone : inclined rectangles,

form curves. An extension of this correlation technique for the 3 D strain measurement is now in labour.

Finally, this correlation technique has to become an another tool for measuring the displacement and/or strain field in certain hard cases. So it is a tool which it is necessary to know the capacities, the power, the rapidity, the limits... to give a result in order to use if it is necessary and for validating, in short time, a numerical or analytical result.

**Index :** Displacement field measurement, correlation technique

## 7. References

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